Image processing and nonparametric regression

Rencontres R BoRdeaux 2012

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Definition:

- noisy image = original image + noise
- Our study: gaussian noise
image: matrix pixel

one pixel

- represent a value of grey level or color level
- is spatially defined by its coordinates \((i, j)\)
- is surrounded with 8 neighbors (vertical, horizontal and diagonal)

numerical measure: Peak Signal to Noise Ratio (PSNR)

\[
PSNR = 10 \times \log_{10}(\frac{d^2}{MSE})
\]

d maximum possible pixel value of the image
MSE mean squared error between the original image and the treated image
quality of reconstruction, in decibel (dB)
well reconstructed image \(\rightarrow 30 \leq \text{PSNR} \leq 40\)
Notations:

- \( y_{ij} \) the grey level at pixel \((i, j)\)
- \( Y \) the grey level of the pixel to be denoised
- \( X \in \mathbb{R}^{10} \) (or more) the vector of explanatory variables

Image denoising as a regression problem.
Non parametric regression model:

\[(X_i, Y_i) \in \mathbb{R}^d \times \mathbb{R} \text{ pairs of observations}\]

\[Y = m(X) + \varepsilon\]

\[\hat{Y} = S_\lambda Y\]

Using method Iterative Bias Reduction (IBR):

- developed by Cornillon, Hengartner et Matzner-Løber
- competes with classic techniques (MARS, GAM)
- estimation without constraints of the regression function \(m\)
- \(R\) package \texttt{ibr} available on cran

Problem:

\[n = 262144 \rightarrow \text{smoothing matrix very big}\]
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\(n = 262144 \rightarrow\) smoothing matrix very big
Resolution:

Image partitioning in small-sized regions

Our proposal:

- free itself from the choice of the size of regions
- have a certain freedom of shape
- data dependent regions
- use of \textbf{CART} (Breiman \textit{et al.}, 1984) and more exactly regression trees
- explain grey level $Y$ by coordinates $(i, j)$

Modifications of package \texttt{rpart} (T.M. Therneau \textit{et al.}, 2002):

- application for high-dimension data
- control maximum and minimum region size
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Modifications of package **rpart** (T.M. Therneau *et al.*, 2002):

- application for high-dimension data
- control maximum and minimum region size
Evolution of partitioning via \texttt{rpart}:

- rectangular regions
- detection of the horizontal and/or vertical structures of the image

Characteristics:

- rectangular regions
- detection of the horizontal and/or vertical structures of the image
Regression model for each region:

\[ Y_i = m(X_i) + \epsilon_i, \quad i = 1, \ldots, n, \]

with \( X \in \mathbb{R}^{10} \) the vector of explanatory variables, and \( n \) the size of the region.

- Question: Influence of the size of regions?
- maximum size of regions: 700 pixels
- number of regions: 686
- region no overlapping → visibility of the outlines of regions
- PSNR = 33.22 dB
Rotation:
- Iteration of image partitioning and image denoising via IBR.
- Rotation of plan $IJ$ between every iteration.
- Prediction of a same pixel with different regions.
- Unique prediction: mean of predicted values.
Question: Influence of the number of rotations?
- maximum size of regions: 300 pixels
- number of rotations: 3
- PSNR = 33.75 dB $\rightarrow + 0.5$ dB
Presentation:
- developed by Dabov et al., 2007
- state of art
- **algorithm**: standard deviation of the noise in parameter → strong influence on the result → unknown noise, default standard deviation = 25
- source **Matlab** code available

Comparison BM3D / IBR:
- several images with various noise
- 3 results:
  1. BM3D with good standard deviation of the noise
  2. BM3D with default standard deviation of the noise
  3. IBR, 3 rotations, maximum size = 700
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Comparison BM3D / IBR:
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- 3 results:
  1. BM3D with good standard deviation of the noise
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BM3D with good standard deviation of the noise (red) :
→ IBR (blue) less successful of more or less 2 dB
→ the differences increase with the level of noise
BM3D with default standard deviation of the noise (green) :
→ IBR better for a low level of noise
Performance:

Image denoising by means of the method **IBR** already gives satisfactory results for its entrance to the vast field of image processing.

Current research:

- more freedom of shape in the creation of regions
- implementation of connections between similar regions

Others applications:

- use of the **rpart** partitioning for high-dimension data in regression
- application in chronological series
**Objective**: reconstruct the missing parts of an image by means of the iterative bias reduction

**Picture data base**:
- black and white images and color images
- accent put on the structural reconstruction
**Process:**

1. Pixels treated one by one, from extremities to the center
2. Definition of a region formed by the $d$ available neighbors in a neighborhood of 4 pixels
3. **Data base:** $K$ nearest neighbors $\rightarrow n$ most similar regions
Inpainting

Method and Results

Color images:
- Formed by three different images (Red, Green, Blue)
- Treatment separated by image

(a) Reference
PSNR : 18.84 dB
SSIM : 0.61

(b) Criminisi et al.
PSNR : 16.5 dB
SSIM : 0.46

(c) Wexler et al.
PSNR : 20.10 dB
SSIM : 0.68

(d) Xu and Sun
PSNR : 23.07 dB
SSIM : 0.83

(e) Proposed
PSNR : 25.83 dB
SSIM : 0.88
Inpainting
Method and Results

Conclusion:

- Good structural reconstruction
- No impression of blur
- **Current research**: use of the structural information of the image in the choice of the filling order
- **Others applications**: treatment of missing data, tested at the moment for chronological series
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